THE PROBLEM OF HEAT TRANSFER THROUGH FINNED SURFACES

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The results of an analytic investigation of heat transfer through a finned surface are presented; the limits of applicability of the assumption that the heat-transfer coefficient remains constant over the entire height of the fin are defined; and recommendations are given for calculations in the region in which this assumption is not applicable.

In the great majority of studies [1-4] devoted to the question of heat transfer through finned surfaces, boundary conditions of the third kind are usually preferred for the analytic investigation of the process; it is usually assumed that the heat-transfer coefficient α is constant over the entire height of the fin, i.e., α = const.

This assumption is effective from the viewpoint of obtaining an analytic solution of the problem, but it is not sufficiently justified for a number of cases of practical importance that use finned heat-transfer surfaces (surfaces with closely spaced fins past which there is a flow with large Re values). Actually, there are a number of factors – the most important of which is the extreme laminarization of the flow at points of inflection – that cause α to vary considerably over the height of the fin. It is impossible to give an a priori law for the variation of α.

Consequently, the problem of heat transfer through a finned surface must be formulated not as a problem with known boundary conditions of the third kind but as a problem that reduces to the simultaneous solution of equations for heat conduction in the wall and for convective transfer of heat in the liquid (or gas) flowing past the fin. The law governing the variation of α along the height of the fin is found by solving this problem.

We give below the results of an approximate solution of the problem of heat transfer through a finned surface, on the basis of which it is possible to determine the limits of applicability of the assumption that α = const.

We consider a plane one-sided finned wall (Fig. 1), past which flows a hydraulically and thermally stabilized turbulent stream of liquid (or gas). The thermophysical properties of the fluid are assumed to be independent of temperature. The notation for the geometric parameters of the wall (the fins are of constant cross section) and the directions of the coordinate axes are shown in Fig. 1.

In order to construct a hydraulic model, we used Prandtl's hypothesis concerning the structure of the turbulent flow, i.e., we assumed that in the rectangular channels formed by the fins there was a turbulent core and a laminar underlayer on the solid surfaces (Fig. 1). We further assumed that there was no overflow of liquid by way of the free surface at the top of the fins and that there was no friction on the surface.

To solve the problem, we must have distribution profiles for the velocities and temperatures in the liquid.

We assumed that in the turbulent core the distribution of velocities and temperatures is represented by a fractional-power function, with exponent 1/7 (Prandtl, Eckert, et al. [3, 5]).

The distribution of velocities in the laminar underlayer is described by a cubical parabola [1, 3, 5].

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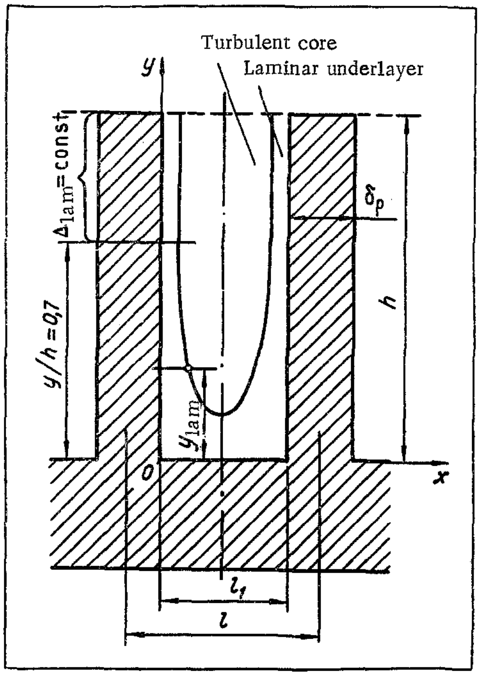

Fig. 1. Finned surface and hydraulic model of flow.

For the laminar underlayer we assumed that the density of the heat flux is constant along a thermal streamline and is equal to the heat flux density on the surface of the fin.

The length of a thermal streamline within the limits of the laminar underlayer (the thickness of the underlayer) can be estimated if we assume that the heat flow is directed transversely across the underlayer, i.e., the thermal streamlines are normal to the boundaries of the flow.

Thus, the thickness of the laminar underlayer was defined as the length of a broken line consisting of two equal line segments each of which is normal to one of the boundaries of the underlayer (the surface of the fin and the line separating the turbulent from the laminar region).

Omitting the intermediate calculations, we give below the relations obtained for the boundary of the laminar underlayer (the line separating the turbulent from the laminar region), the thickness of the laminar underlayer, and the density of the heat flux on the surface of the fin.

The equation of the boundary of the laminar underlayer is as follows:

$$0\leqslant \bar{y}<0.7 \quad \bar{y}_{lam}=\frac{y_{lam}}{h}=\frac{155{,}5}{\mathrm{Re}^{0.875}\bar{x}},$$
$$0.7\leqslant \bar{y}\leqslant 1{,}0 \quad \bar{x}_{lam}=\frac{x_{lam}}{\frac{1}{2}l_1}=\frac{221{,}5}{\mathrm{Re}^{0.875}}. \tag{1}$$

The thickness of the laminar underlayer is:

$$0\leqslant \bar{y}<0.7 \quad \bar{\Delta}_{lam}=\frac{\Delta_{lam}}{\frac{1}{2}l_1}=\left[\frac{\mathrm{Re}^{0.438}}{21.9}+\frac{\mathrm{Re}^{0.875}}{144.5}\left(1-\frac{10.1}{\mathrm{Re}^{0.438}}\right)\bar{y}^{1.2}\right]^{-1}, \tag{2}$$
$$0.7\leqslant \bar{y}\leqslant 1.0 \quad \bar{\Delta}_{lam}=\bar{x}_{lam}.$$

The density of the heat flux on the surface of the fin is:

$$q_f=\frac{\lambda_{liq}}{\frac{1}{2}l_1}F_1(T_f-F_2),$$
$$l_1=l-\delta_f. \tag{3}$$

The dimensionless parameters in formulas (1), (2), and (3) are defined as follows:

$\bar{x}=(x)/(1/2)l_1$, $\bar{y}=y/h$ are dimensionless linear coordinates;

$\mathrm{Re}=(w_{av}2l_1)/\nu$ is the Reynolds number, as determined from the stream velocity for the average flow rate;

$T_f=(t_f-t_*)/(t_{f0}-t_*)$ is a dimensionless expression for the surface temperature of the fin;

$F_1(\mathrm{Re},\bar{y})$, $F_2(\mathrm{Re},\bar{y})$ are known functions of Re and $\bar{y}$.

Setting (3) equal to the expression for q_f obtained from the equation for heat conduction in a fin as given in a number of studies [1, 2], we have the differential equation

$$\frac{d^2T_f}{d\bar{y}^2}-\Phi F_1T_f=-\Phi F_1F_2. \tag{4}$$

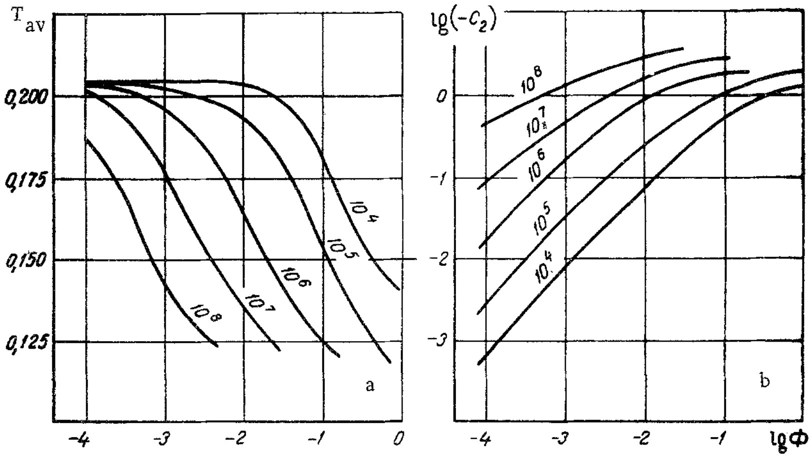

Fig. 2. a) Curves for determining the average temperature of the liquid (or gas); b) curves for determining the total amount of heat removed from the fin (the figures next to the curves represent Reynolds numbers).

It follows from Eq. (4) that the temperature distribution along the height of the fin is determined only by the two dimensionless quantities Re and Φ, which considerably simplifies the analysis of our solution. The quantity Φ combines the thermophysical characteristics of the liquid and the wall material and the geometric parameters of the surface:

$$\Phi = \frac{\lambda_{liq}}{\lambda_f} \frac{2h^2}{\frac{1}{2} l_1 \delta_f}.$$

The solution of Eq. (4) was obtained as the sum of a power series. This problem was solved numerically on an electronic computer.

The values of Re and Φ were varied over the ranges $10^4 < Re < 10^8$, $10^{-4} < \Phi < 1$, which include the ranges found in the operation of present-day heat-transfer devices.

Using the results of calculations for the fin temperatures, we investigated the local and average heat transfer. The local heat-transfer coefficient is given by the formula:

$$\alpha(y) = \frac{q_f(y)}{t_f(y) - t_{av}} \tag{5}$$

or, in dimensionless form:

$$\mathrm{Nu} = \frac{\alpha(\bar{y})\, 2l_1}{\lambda_{liq}} = \frac{4F_1(T_f - F_2)}{T_f - T_{av}}, \tag{5a}$$

where $T_{av} = (t_{av} - t_*)/(t_{f0} - t_*)$ is the dimensionless average calorimetric temperature of the liquid (the dependence of which on Re and Φ is shown in Fig. 2a).

The heat-transfer coefficient averaged over the height of the fin is defined as

$$\alpha_{av} = \frac{Q_f}{2h\,(t_f - t_{av})_{av}}, \tag{6}$$

where $(t_f - t_{av})_{av}$ is the temperature difference between fin and fluid, averaged over the height of the fin.

For the value of Q_f we obtained the expression

$$Q_f = \frac{t_{f0} - t_{av}}{1 - T_{av}} \frac{\lambda_f \delta_f}{h} (-C_2). \tag{7}$$

The coefficient C_2 in formula (7) is the value of the derivative $(dT_f/d\bar{y})_{\bar{y}=0}$.

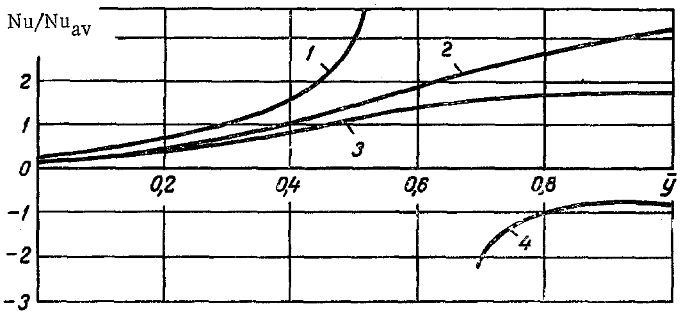

Fig. 3. Distribution of local heat exchange along the height of the fin for Re = 10^5 ($\Phi = 1$ for curves 1 and 4; $\Phi = 10^{-1}$ for curve 2; $\Phi \le 10^{-2}$ for curve 3).

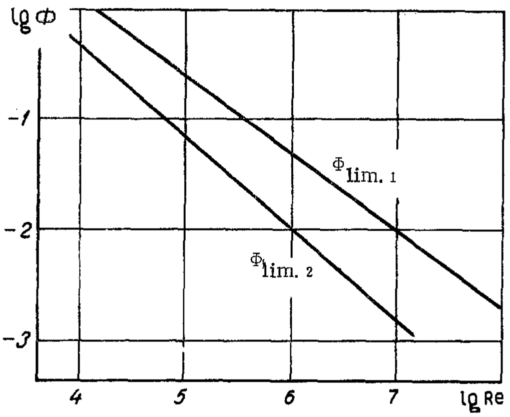

Fig. 4. Relationship between the Reynolds number Re and the limiting values of Φ.

The value of the coefficient C_2 was found from calculations of the variation in temperature along the height of the fin; the variation of C_2 as a function of Re and Φ is shown in Fig. 2b.

Substituting (7) into (6) and making some transformations, we obtain the following expression for Nu_{av}, the Nusselt number averaged over the height of the fin:

$$Nu_{av} = \frac{4(-C_2)}{\Phi} \frac{1}{T_{f.av} - T_{av}}. \qquad (6a)$$

Figure 3 shows the results of calculations for the variation of local heat exchange with height along the fin for Re = 10^5 and different values of Φ.

The heat exchange is very unevenly distributed along the height of the fin: at the base of the fin the local value of Nu may be only a few percent of Nu_{av}; as $\bar{y}$ increases, so does Nu, and at the top of the fin the local Nu is several times as great as Nu_{av}. For a fixed $\bar{y}$ value, Nu (or α) approaches infinity as Re and Φ increase, but for larger values of $\bar{y}$ it falls into the negative region.

This means that the local value of t_f becomes less than the average calorimetric temperature of the liquid, and the usual definition of α given in (5) is inapplicable in this region. This result agrees with the conclusions arrived at by Chapman [6] and T. L. Perel'man (ITMO AN BSSR).

The limiting values of $\Phi_{lim.1}$ for different Reynolds numbers Re, with α remaining constant over the entire height of the fin, are shown in the form of graphs in Fig. 4.

A comparison of the calculated results for the averaged heat exchange with the experimental data of other authors [4, 7] for heat exchange in rectangular channels with one-sided heating shows great agreement. We found that for Re $\gg 10^4$, the calculation of the average heat exchange based on fractional-power functions obtained by generalizing the experimental data on heat exchange in channels with isothermal surfaces gave us results which were too high (by more than 20%).

The limits of applicability of the assumption α = const were determined on the basis of the requirement that the calculated value of the temperature at the base of a fin, which is the maximum temperature of the finned side, should yield values differing by no more than 10% when calculated by the two procedures being compared.

The following expression is known for Q_f when α = const [1]:

$$Q_f = (t_{f0} - t_{av}) \lambda_f \delta_f m \, \mathrm{th}(mh),$$
$$m = \sqrt{\frac{2\alpha}{\delta_f \lambda_f}} = \frac{1}{2h} \sqrt{\Phi \, \mathrm{Nu}}. \qquad (8)$$

It should be noted that when the calculations are made by the method using the assumption α = const, the fluid temperature t_{av} is given as an integral part of the boundary conditions.

Since the heat flux given off by a fin into the cooling medium is independent of the method by which the heat transfer is calculated, it follows that by equating the right sides of the expressions (7) and (8), we can obtain a relation for comparing the temperatures t_{f0} obtained by the two methods under comparison:

$$\frac{t_{f0}-t_{av}}{[t_{f0}-t_{av}]_{\alpha=\mathrm{const}}}=\frac{\frac{1}{2}\sqrt{\Phi\,\mathrm{Nu}_{av}}\,\mathrm{th}\left(\frac{1}{2}\sqrt{\Phi\,\mathrm{Nu}_{av}}\right)}{(-C_2)}(1-T_{av}). \tag{9}$$

For small values of Re and Φ the temperatures under comparison are in good agreement. However, as Re and Φ increase, the temperature t_{f0} calculated on the assumption that α = const will be considerably lower than the temperatures t_{f0} found by calculations taking account of the variation of α.

Thus, to disregard the variation of heat exchange with height along the fin may result in the designed heat exchanger being unworkable, since the calculated value of the maximum temperature will be lower than the true value.

The graph in Fig. 4 shows how the limiting value of the complex $\Phi_{lim.2}$ varies with Re. For $\Phi < \Phi_{lim.2}$ the heat transfer through the finned surface may be calculated by methods using the assumption α = const (the error in the temperature calculation will not exceed 10%).

NOTATION

x, y	are the linear coordinates;
h	is the height of fin;
l	is the pitch of fins (distance between fins along the axis);
δ_f	is the thickness of fin;
x_{lam}, y_{lam}	are the coordinates of the boundary of the laminar underlayer;
Δ_{lam}	is the thickness of the laminar underlayer;
w_{av}	is the stream velocity for average flow rate;
Q_f	is the heat flux removed from the surface of the fin;
q_f	is the density of heat flux on the surface of the fin;
α, α_{av}	are the heat-transfer coefficient, local value and value averaged over the height of the fin, respectively;
λ_{liq}, λ_f	are the thermal conductivity values for the liquid (gas) and the fin material;
ν	is the kinematic viscosity of the liquid (gas);
t_{av}, t_*	are the temperature of liquid (gas), average calorimetric value and value along the interfin axis at the level of the top of each fin, respectively;
t_f, t_{f0}	are the fin temperatures (locally along the fin and at the base);
Re	is the Reynolds number;
Nu, Nu_{av}	are the Nusselt number, as determined from the local value of α and the average value α_{av}, respectively;
Φ	is the dimensionless quantity;
T_{av}	is the dimensionless average calorimetric temperature of the liquid (gas);
$T_f, T_{f.av}$	are the dimensionless temperature of the fin, local value and value averaged over the height, respectively.

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